EJERCICIO (29:42)

Demostrar que:

$$\left\langle 0 \left| T\left\{ (a_c(\infty) - a_c(-\infty))(a_d(\infty) - a_d(-\infty))\left(a_a^\dagger(-\infty) - a_a^\dagger(\infty)\right)\left(a_b^\dagger(-\infty) - a_b^\dagger(\infty)\right)\right\}\right|0\right\rangle$$
$$= \left\langle 0 \left| T\{a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)\}\right|0\right\rangle$$

Desarrollamos:

$$\left(a_c(\infty) - a_c(-\infty)\right)\left(a_d(\infty) - a_d(-\infty)\right)\left(a_a^\dagger(-\infty) - a_a^\dagger(\infty)\right)\left(a_b^\dagger(-\infty) - a_b^\dagger(\infty)\right) =$$

$$= \left(a_c(\infty)a_d(\infty) - a_c(\infty)a_d(-\infty) - a_c(-\infty)a_d(\infty) + a_c(-\infty)a_d(-\infty)\right)\left(a_a^\dagger(-\infty)a_b^\dagger(-\infty)\right.$$
$$\left. - a_a^\dagger(-\infty)a_b^\dagger(\infty) - a_a^\dagger(\infty)a_b^\dagger(-\infty) + a_a^\dagger(\infty)a_b^\dagger(\infty)\right) =$$

$$= a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty) - a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(\infty) - a_c(\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)$$
$$+ a_c(\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty) -$$

$$-a_c(\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty) + a_c(\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(\infty) + a_c(\infty)a_d(-\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)$$
$$- a_c(\infty)a_d(-\infty)a_a^\dagger(\infty)a_b^\dagger(\infty) -$$

$$-a_c(-\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty) + a_c(-\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(\infty) + a_c(-\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)$$
$$- a_c(-\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty) +$$

$$+a_c(-\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty) - a_c(-\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(\infty) - a_c(-\infty)a_d(-\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)$$
$$+ a_c(-\infty)a_d(-\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)$$

Aplicamos la ordenación temporal a cada término de la expresión anterior:

1) $T\{a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)\} = a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)$
2) $T\{a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(\infty)\} = a_c(\infty)a_d(\infty)a_b^\dagger(\infty)a_a^\dagger(-\infty)$
3) $T\{a_c(\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)\} = a_c(\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)$
4) $T\{a_c(\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)\} = a_c(\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)$
5) $T\{a_c(\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)\} = a_c(\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)$
6) $T\{a_c(\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(\infty)\} = a_c(\infty)a_b^\dagger(\infty)a_d(-\infty)a_a^\dagger(-\infty)$
7) $T\{a_c(\infty)a_d(-\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)\} = a_c(\infty)a_a^\dagger(\infty)a_d(-\infty)a_b^\dagger(-\infty)$
8) $T\{a_c(\infty)a_d(-\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)\} = a_c(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)a_d(-\infty)$
9) $T\{a_c(-\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)\} = a_d(\infty)a_c(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)$
10) $T\{a_c(-\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(\infty)\} = a_d(\infty)a_b^\dagger(\infty)a_c(-\infty)a_a^\dagger(-\infty)$
11) $T\{a_c(-\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)\} = a_d(\infty)a_a^\dagger(\infty)a_c(-\infty)a_b^\dagger(-\infty)$
12) $T\{a_c(-\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)\} = a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)a_c(-\infty)$
13) $T\{a_c(-\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)\} = a_c(-\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)$

14) $T\{a_c(-\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(\infty)\} = a_b^\dagger(\infty)a_c(-\infty)a_d(-\infty)a_a^\dagger(-\infty)$

15) $T\{a_c(-\infty)a_d(-\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)\} = a_a^\dagger(\infty)a_c(-\infty)a_d(-\infty)a_b^\dagger(-\infty)$

16) $T\{a_c(-\infty)a_d(-\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)\} = a_a^\dagger(\infty)a_b^\dagger(\infty)a_c(-\infty)a_d(-\infty)$

Aplicamos la relación de conmutación a los 16 términos, $[a_k , a_q^\dagger] = 0$ para k ≠ q:

1) $a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)$

2) $a_c(\infty)a_d(\infty)a_b^\dagger(\infty)a_a^\dagger(-\infty) = a_c(\infty)a_b^\dagger(\infty)a_d(\infty)a_a^\dagger(-\infty) = a_b^\dagger(\infty)a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)$

3) $a_c(\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty) = a_c(\infty)a_a^\dagger(\infty)a_d(\infty)a_b^\dagger(-\infty) = a_a^\dagger(\infty)a_c(\infty)a_d(\infty)a_b^\dagger(-\infty)$

4) $a_c(\infty)a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty) = a_a^\dagger(\infty)a_c(\infty)a_d(\infty)a_b^\dagger(\infty) = a_a^\dagger(\infty)a_c(\infty)a_d(\infty)a_b^\dagger(\infty)$

5) $a_c(\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty) = a_c(\infty)a_a^\dagger(-\infty)a_d(-\infty)a_b^\dagger(-\infty) = a_c(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)a_d(-\infty)$

6) $a_c(\infty)a_b^\dagger(\infty)a_d(-\infty)a_a^\dagger(-\infty) = a_c(\infty)a_b^\dagger(\infty)a_a^\dagger(-\infty)a_d(-\infty)$

7) $a_c(\infty)a_a^\dagger(\infty)a_d(-\infty)a_b^\dagger(-\infty) = a_c(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)a_d(-\infty)$

8) $a_c(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)a_d(-\infty)$

9) $a_d(\infty)a_c(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty) = a_d(\infty)a_a^\dagger(-\infty)a_c(-\infty)a_b^\dagger(-\infty) = a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)a_c(-\infty)$

10) $a_d(\infty)a_b^\dagger(\infty)a_c(-\infty)a_a^\dagger(-\infty) = a_d(\infty)a_b^\dagger(\infty)a_a^\dagger(-\infty)a_c(-\infty)$

11) $a_d(\infty)a_a^\dagger(\infty)a_c(-\infty)a_b^\dagger(-\infty) = a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)a_c(-\infty)$

12) $a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)a_c(-\infty)$

13) $a_c(-\infty)a_d(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty) = a_c(-\infty)a_a^\dagger(-\infty)a_d(-\infty)a_b^\dagger(-\infty) =$
    $a_c(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)a_d(-\infty)$

14) $a_b^\dagger(\infty)a_c(-\infty)a_d(-\infty)a_a^\dagger(-\infty) = a_b^\dagger(\infty)a_c(-\infty)a_a^\dagger(-\infty)a_d(-\infty)$

15) $a_a^\dagger(\infty)a_c(-\infty)a_d(-\infty)a_b^\dagger(-\infty) = a_a^\dagger(\infty)a_c(-\infty)a_b^\dagger(-\infty)a_d(-\infty)$

16) $a_a^\dagger(\infty)a_b^\dagger(\infty)a_c(-\infty)a_d(-\infty)$

Contraemos estos términos en el vacío: $\langle 0|T\acute{e}rmino|0\rangle$ y teniendo en cuenta que:

  a) $a|0\rangle = 0$
  b) $\langle 0|a^\dagger = 0$

Se obtiene que los términos 2 a 4 se anulan por (b), los términos 5 a 16 se anulan por (a), quedando sólo el primero de ellos:

1) $\langle 0|a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)|0\rangle = \langle 0|T\{a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)\}|0\rangle \neq 0$

2) $\langle 0|a_b^\dagger(\infty)a_c(\infty)a_d(\infty)a_a^\dagger(-\infty)|0\rangle = 0$

3) $\langle 0|a_a^\dagger(\infty)a_c(\infty)a_d(\infty)a_b^\dagger(-\infty)|0\rangle = 0$

4) $\langle 0|a_a^\dagger(\infty)a_c(\infty)a_d(\infty)a_b^\dagger(\infty)|0\rangle = 0$

5) $\langle 0|a_c(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)a_d(-\infty)|0\rangle = 0$

6) $\langle 0|a_c(\infty)a_b^\dagger(\infty)a_a^\dagger(-\infty)a_d(-\infty)|0\rangle = 0$

7) $\langle 0|a_c(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)a_d(-\infty)|0\rangle = 0$

8) $\langle 0|a_c(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)a_d(-\infty)|0\rangle = 0$

9) $\langle 0|a_d(\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)a_c(-\infty)|0\rangle = 0$

10) $\langle 0|a_d(\infty)a_b^\dagger(\infty)a_a^\dagger(-\infty)a_c(-\infty)|0\rangle = 0$

11) $\langle 0|a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(-\infty)a_c(-\infty)|0\rangle = 0$

12) $\langle 0|a_d(\infty)a_a^\dagger(\infty)a_b^\dagger(\infty)a_c(-\infty)|0\rangle = 0$

13) $\langle 0|a_c(-\infty)a_a^\dagger(-\infty)a_b^\dagger(-\infty)a_d(-\infty)|0\rangle = 0$

14) $\langle 0 | a_b^\dagger(\infty) a_c(-\infty) a_a^\dagger(-\infty) a_d(-\infty) | 0 \rangle = 0$

15) $\langle 0 | a_a^\dagger(\infty) a_c(-\infty) a_b^\dagger(-\infty) a_d(-\infty) | 0 \rangle = 0$

16) $\langle 0 | a_a^\dagger(\infty) a_b^\dagger(\infty) a_c(-\infty) a_d(-\infty) | 0 \rangle = 0$

De este modo se comprueba que:

$$\boxed{\begin{array}{c} \langle 0 | T\left\{ \left(a_c(\infty) - a_c(-\infty)\right)\left(a_d(\infty) - a_d(-\infty)\right)\left(a_a^\dagger(-\infty) - a_a^\dagger(\infty)\right)\left(a_b^\dagger(-\infty) - a_b^\dagger(\infty)\right)\right\} | 0 \rangle \\ = \langle 0 | T\{a_c(\infty) a_d(\infty) a_a^\dagger(-\infty) a_b^\dagger(-\infty)\} | 0 \rangle \end{array}}$$